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LETTERS TO THE EDITORS

THE CONSTANT PROPERTY TURBULENT BOUNDARY LAYER WITH INJECTION; A REANALYSIS OF SOME EXPERIMENTAL RESULTS

(*Received* 11 *November* 1969)

NOMENCLATURE

a, b, constants in equation (3);
c_f, skin-friction coefficient;
F, injection parameter $(= \rho_w v_w/\rho_\infty u_\infty)$;
R_x, Reynolds number based on distance along plate;
R_θ, Reynolds number based on momentum thickness;
R'_θ, modified Reynolds number defined in equation (2);
u, local velocity;
U_∞, free stream velocity;
v_w, injection velocity normal to wall;
θ, momentum thickness;
ρ_w, density of injected fluid;
ρ_∞, density in free stream;
ν, kinematic viscosity.

NB. Simpson uses $\dot{m}''$ for $\rho_w v_w$, G for $\rho_\infty U_\infty$ and does not have a symbol for the ratio, $\rho_w v_w/\rho_\infty U_\infty$.

1. INTRODUCTION

IN RECENT papers McQuaid [1] and Simpson *et al.* [2, 3] have published the results of two extensive experimental investigations of the turbulent boundary layer on a porous surface with fluid injection into the layer. In both sets of experiments boundary layer profiles were measured at various stations along the surface and the skin friction was determined from the measured growth of the momentum thickness. Simpson* in the discussion of McQuaid's results points out that "Examination of his (McQuaid's) data in $c_f/2$ vs. R_θ coordinates, as proposed by Rotta, reveals an apparent velocity dependence which was not expected.... The existence of a velocity dependence in McQuaid's results, even though inside the quoted uncertainty band, suggests that the structure of the apparatus might, in some way, have affected the results". For this reason Simpson did not compare his results with McQuaid's. In the light of these comments the present author has reanalysed both McQuaid's and Simpson's results. As a result of this reanalysis it is found that the discrepancy in McQuaid's results is probably caused by McQuaid's method of analysis and curve fitting, rather than to any basic fault in the experimental apparatus.

It was also found that Simpson's data can also yield different values of the skin friction by different curve-fitting methods. As a result the skin-friction coefficient based on Simpson's data show a much larger scatter than the uncertainty bands quoted in his report. However, in spite of this scatter it was found that there was a significant difference between the results obtained from the two experiments. In particular McQuaid's results tend to give lower skin-friction values than do Simpson's results.

Although we are only concerned here with boundary layers with zero pressure gradient homogeneous injection, the papers mentioned above do contain data on other cases, together with full experimental details of the two tests.

2. ANALYSIS

In both sets of experiments skin-friction coefficients were determined from the measured boundary layer growth by means of the momentum integral equation. For a two-dimensional boundary layer in zero pressure gradient this equation takes the form

$$\frac{c_f}{2} = \frac{d\theta}{dx} - F = \frac{dR_\theta}{dR_x} - F. \tag{1}$$

McQuaid used equation (1) directly to determine his skin friction. He first plotted θ against x for one tunnel speed and blowing rate, and fitted what he considered to be the best curve through the points. He then found the slope of this curve, i.e. $d\theta/dx$ at different points along the surface. On the other hand Simpson re-wrote equation (1) in the form

$$\frac{c_f}{2} = \frac{d}{dR_x}\left(R_\theta - \int_0^{R_x} F\, dR_x\right),$$
$$= \frac{dR'_\theta}{dR_x} \text{(say)}, \tag{2}$$

and then assumed that R'_θ was related to R_x by

$$R'_\theta = aR_x^b, \tag{3}$$

so that

$$\frac{c_f}{2} = \frac{dR'_\theta}{dR_x} = abR_x^{b-1}. \tag{4}$$

* In the rest of this paper we shall refer to the paper of Simpson *et al.* as Simpson.

The constants a and b were determined for any given blowing rate by plotting R'_θ against R_x on log–log paper, and in the analysis it was assumed that virtual origin was always at the leading edge, i.e. it was unchanged by blowing.

Figure 1 shows the skin-friction coefficients as determined

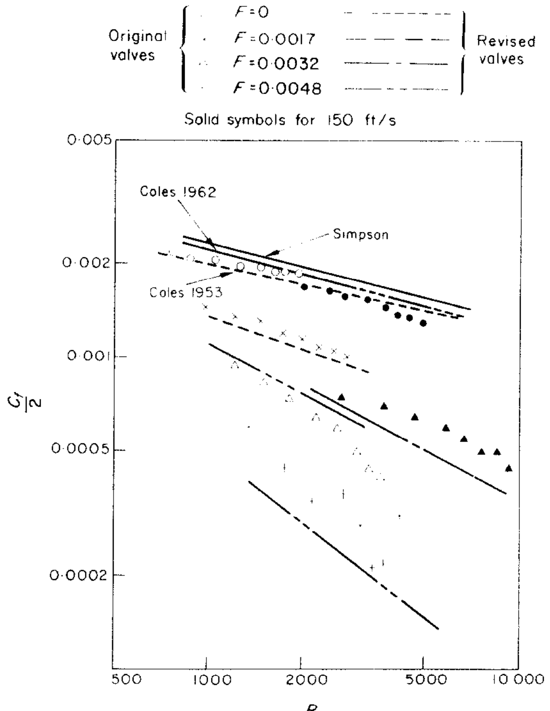

FIG. 1.

by McQuaid, together with the zero-blowing results of Simpson. The results for $F = 0{\cdot}0032$ at the two tunnel speeds do not agree suggesting that $c_f/2$ is not only a function of R_θ and F, but also depends on U_∞. Also the zero injection results from the two experimental investigations are different, although they are both in fair agreement with the results suggested by Coles [4, 5]. In order to study the discrepancy in the results at $F = 0{\cdot}0032$ the measurements† were replotted as R'_θ against R_x where x is the distance from the leading edge. The results for the two speeds lie in two bands and it is believed that the step between the two bands is associated with small errors in the injection rate. It is possible to draw a number of straight lines through the data (see Fig. 2) but the step in skin-friction is least if we fit curve 'a' to the data at 50 ft/s and curve 'b' to the data at 150 ft/s. However, it seems that McQuaid fitted curves of type 'b' to both sets of data and also gave more weight to the three points at the downstream end of the test section at 50 ft/s. The revised skin-friction coefficients are plotted in Fig. 1 together with the revised values for other blowing rates.

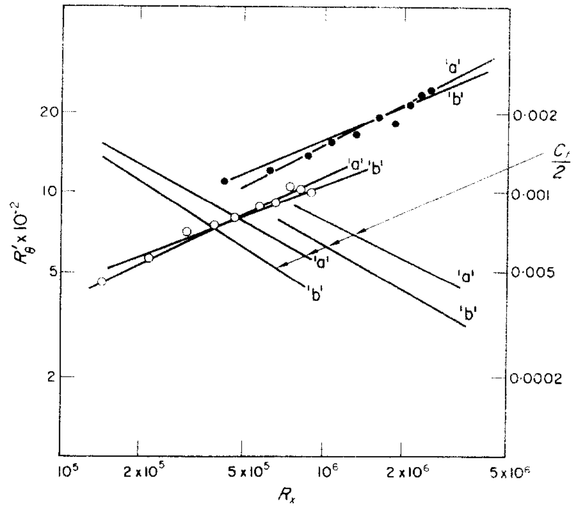

FIG. 2.

In the reanalysis of Simpson's data it was found that although the straight line suggested by Simpson was a good fit in log–log coordinates in the middle of the test range ($4 \times 10^5 \leqslant R_x \leqslant 1{\cdot}6 \times 10^6$) the measured points at both ends of the range tended to lie above the line. It was also found that a better straight line fit to the whole data could be obtained by plotting $\log R'_\theta$ against $\log(R_x + 2 \times 10^5)$. This is illustrated in Fig. 3 where all the results are plotted in natural coordinates, together with the two suggested fits. In general the fit suggested by the present author gives much less variation of $c_f/2$ with R_x than the fit used by Simpson (see Fig. 4). In particular the results for zero injection at 40 ft/s lie on a straight line in physical coordinates, giving a constant skin friction coefficient of 0·0039 for $1{\cdot}3 \times 10^5 \leqslant R_x \leqslant 1{\cdot}8 \times 10^6$, as compared with values quoted by Simpson which range from 0·0055 at $R_x = 1{\cdot}3 \times 10^5$ to 0·0033 at $R_x = 2 \times 10^6$. In the light of this straight line fit in physical coordinates it is difficult to believe Simpson's claim of ± 5 per cent accuracy on his quoted skin-friction coefficients, or the claim that "Friction factors obtained by the momentum integral equation method agreed within 2 per cent of the expected relation

$$\frac{c_f}{2} = 0{\cdot}0296\, R_x^{-0{\cdot}2}$$

† In calculating R'_θ the possible variation of ± 10 per cent in the porosity of injection surface has been ignored, and the injection rate (as suggested by a recent calibration) has been taken as $0{\cdot}7F$ at the beginning of the plate rising to a uniform value of $1{\cdot}02F$ 3 in. from the beginning of the plate, where F is McQuaid's mean value.

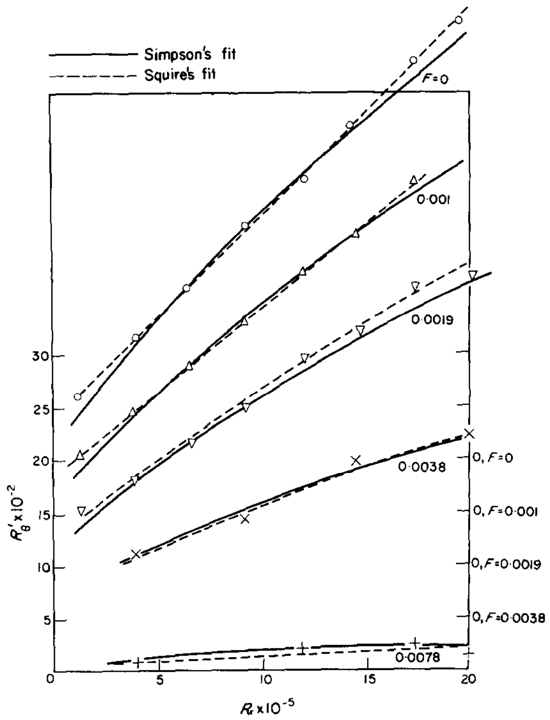

FIG. 3.

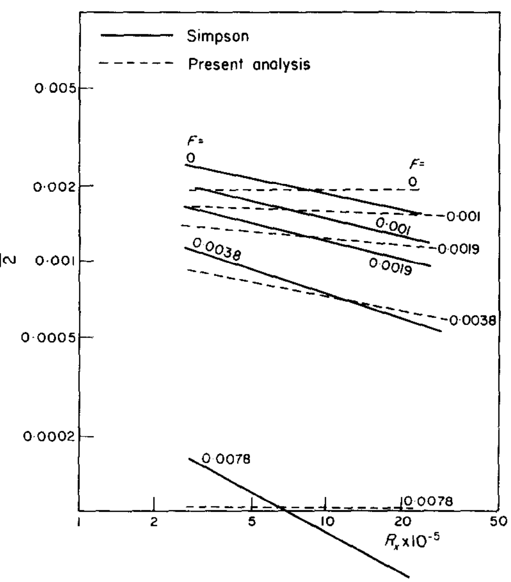

FIG. 4.

for

$$4 \times 10^5 \leqslant R_x \leqslant 2 \times 10^6."$$

In fact the present author believes that, since one set of his measurements for zero blowing gave an almost linear variation R_θ with R_x for the full test length, all the results with injection may be in error.

It should be noted that although Simpson's results at 80 ft/s with zero injection appear to support his skin-friction results, the values of R_x for these results were chosen to make the points lie on his fitted curve.

Simpson also obtained skin-friction coefficients by a sublayer method and obtained results in agreement with his momentum analysis. This analysis is based on the fact that near the wall the profiles should tend to

$$\frac{u}{U_\infty} = \frac{c_f}{2F}\left[\exp\left(F\frac{U_\infty y}{\nu}\right) - 1\right], \tag{5}$$

or

$$\frac{u}{U_\infty} = \frac{c_f}{2}\frac{U_\infty y}{\nu} \quad \text{for} \quad F = 0. \tag{6}$$

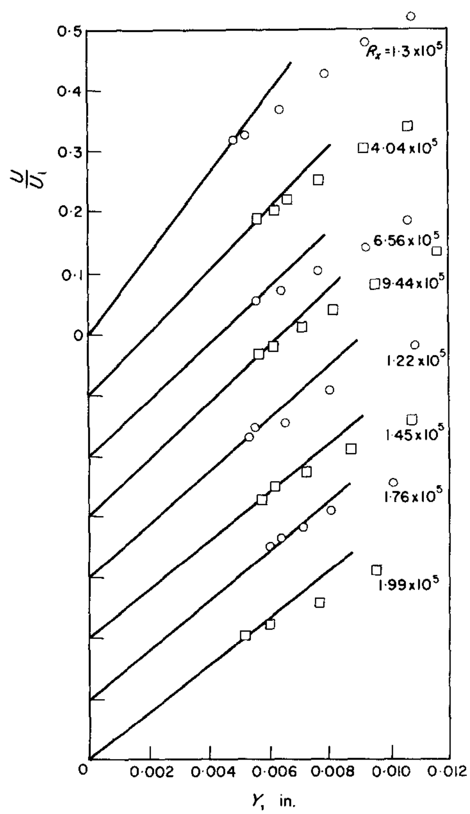

FIG. 5.

Thus for given blow rate and unit Reynolds number these equations can be used to find c_f from measured values of y and u/U_∞, provided the points are within the sublayer. Simpson used the first measured point to find c_f, and also found that "In many cases, several consecutive y-stations produced the same value of $c_f/2$, lending credence to the method". Figure 5 shows the velocity profiles measured near the wall with zero blowing together with the sub-layer profile based on Simpson's sublayer skin friction. In general the measured points lie on lines which intersect the sublayer profile rather than blend into it. This suggests that the measured points do not lie in the sublayer, and the fact that Simpson gets sensible skin-frictions values probably implies some cancellation of errors. In particular it is easy to see that a displacement effect of 0·001 in. (0·1 times the height of the probe) could make the results blend into a sublayer profile with the right skin friction.

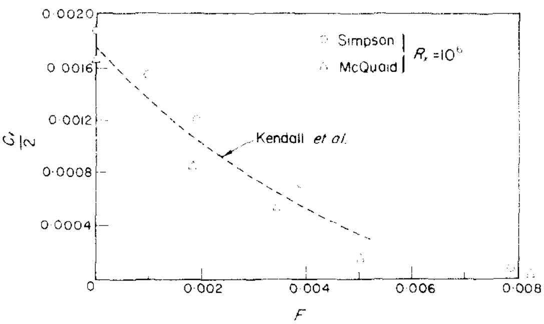

FIG. 6.

The skin friction results of the two sets of experiments are compared in Fig. 6 at a Reynolds number of $R_x = 10^6$. This figure also included the values obtained by Kendall *et al.* [6] from an extensive survey of all the measurements made at M.I.T. under H. S. Mickley. As will be seen Simpson's results are much higher than those of McQuaid with Kendall's results occupying a mean position.

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REPLY TO THE COMMENTS BY DR. Y. R. MAYHEW ON THEORETICAL STUDY OF LAMINAR FILM CONDENSATION OF FLOWING VAPOUR

IT WAS of great interest for us to read the comments [1] by Dr. Y. R. Mayhew from the University of Bristol on our paper [2].

It was shown in papers [2, 3] that a transverse mass flow across the vapour–liquid interface due to phase change is the dominant factor in the hydrodynamics of film condensation for vapour flowing longitudinally over a flat plate. Even at negligible rates of phase change the presence of the above-mentioned flow excludes the occurence of the turbulent boundary layer in the vapour flow, and for condensation processes it develops such a flow at which interfacial shear is mainly determined by the momentum trans-